Mathematics 31

Information Package

January 1995

QA
14
C22
A3
A315
1995
gr.12
info. pkg.

CURRGDHT

Alberta
EDUCATION
Curriculum Standards Branch

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

TABLE OF CONTENTS

PAGE

INTRODUCTION 1

Scope and Structure 1
Sequencing Mathematics 30 and Mathematics 31 2

RESOURCES 3

Chart 1 through Chart 4 3
Chart 5 through Chart 8 4

Chart 1: Correlation for Stewart et al., *Calculus: A First Course* 5
Chart 2: Correlation for Reid et al., *Calculus: A Problem-Solving Approach* 6
Chart 3: Correlation for Stewart, *Single Variable Calculus* 7
Chart 4: Correlation for ACCESS Network, *Catch 31* Videos 8
Chart 5: Reverse Correlation for Stewart et al., *Calculus: A First Course* 9
Chart 6: Reverse Correlation for Reid et al., *Calculus: A Problem-Solving Approach* 10
Chart 7: Reverse Correlation for Stewart, *Single Variable Calculus* 11
Chart 8: Reverse Correlation for ACCESS Network, *Catch 31* Videos 12

TECHNOLOGY AND MATHEMATICS 31 13

INTRODUCTION

SCOPE AND STRUCTURE

The Mathematics 31 course is designed to bridge the gap between the Mathematics 10–20–30 program sequence and the calculus course sequences offered by post-secondary institutions. The course emphasizes the theoretical and practical developments of topics in the algebra of functions, trigonometry, differential calculus and integral calculus up to a standard acceptable for entry into all first-year programs in mathematics, science, engineering and business.

The specific purposes of the Mathematics 31 Course of Studies are as follows:

- to develop an understanding of the algebra of functions and transformations, together with their graphs, and to apply these understandings in different areas of mathematics
- to develop a fluency in algebraic computations involving rational expressions, inequalities, absolute values and trigonometric functions
- to introduce the principal concepts and methods of differential and integral calculus
- to develop skills in problem solving and reasoning, using calculus concepts and procedures as the context
- to apply the methods of calculus to various simple applications in the physical and biological sciences, in engineering, in business and in the social sciences.

The Mathematics 31 course is organized in a required–elective format, with eight elective components available for use. Most students will do one or two electives, but those students attempting to satisfy the requirements of such programs as International Baccalaureate (Subsidiary Level), Advanced Placement Calculus (AB Option), Ontario Academic Course (Calculus), and first-year Collège d'enseignement général et professionel (Québec) need to do up to four electives.

Most students who are taking Mathematics 31 to satisfy the requirements for admission to programs in Alberta post-secondary institutions will take one or two electives. In general, the elective on the calculus of exponential and logarithmic functions has the greatest range of applicability, as most biological applications depend on the exponential function, and all Alberta post-secondary institutions introduce the exponential and logarithmic functions at an early point in their calculus courses.

The four sections of the required component are as follows:

- precalculus and limits
- derivatives and derivative theorems
- applications of derivatives
- integrals, integral theorems and integral applications.

The eight possible units available in the elective component are as follows:

- calculus of exponential and logarithmic functions
- numerical methods
- volumes of revolution
- applications of calculus to physical sciences and engineering

- applications of calculus to biological sciences
- applications of calculus to business and economics
- calculus theorems
- further methods of integration.

These electives may be taught as separate units of instruction or integrated with the relevant sections of the required component.

SEQUENCING MATHEMATICS 30 AND MATHEMATICS 31

In most cases, students take Mathematics 31 after they have completed Mathematics 30, and such a sequence is highly recommended. However, there will be some circumstances where students take Mathematics 30 and Mathematics 31 concurrently, whether over an academic year or in a single semester. In these cases, adjustments may need to be made to the sequence of instruction for both Mathematics 30 and Mathematics 31. Some suggested adjustments follow.

In Mathematics 30, the units on polynomial functions and trigonometric functions should be done early in the course, with the units on exponential and logarithmic functions, and quadratic relations, done in the middle of the course. The discrete mathematics units on sequences and series, permutations and combinations, and on statistics, should be taught at the end of the Mathematics 30 course.

In Mathematics 31, the precalculus and limits section should be taught on an as needed basis, with the initial work on limits, derivatives and derivative theorems based on algebraic functions only. The precalculus and calculus of trigonometric functions should be done either after derivatives and derivative theorems or after applications of derivatives, and should be done after the unit on trigonometric functions has been completed in Mathematics 30. If the elective on the calculus of exponential and logarithmic functions, or on the applications of calculus to biological sciences, is selected, it should be done near the end of Mathematics 31 and after the unit on exponential and logarithmic functions has been covered in Mathematics 30.

In some schools, there is a mixture of full-year and semestered courses in Grade 12 mathematics. In these cases, it is preferable that Mathematics 31 be the full-year course, with Mathematics 30 in the first semester. It is acceptable that Mathematics 30 be the full-year course, with Mathematics 31 in the second semester. It is not advisable to have Mathematics 31 as a full-year course, with Mathematics 30 in the second semester.

RESOURCES

The choice of appropriate resources is made at the school level.

There are three choices of text resources for individual students.

- Stewart, James, Thomas M. K. Davison and Bryan Ferroni. *Calculus: A First Course.* Toronto, ON: McGraw-Hill Ryerson Limited, 1989.

  ISBN 0075496011 LRDC Product Order Number 262006

- Reid, Neal, Douglas Mark, Phil Feldman, Russell Garrett, Joseph Geiser and David Handley. *Calculus: A Problem-Solving Approach.* Toronto, ON: John Wiley & Sons Canada Limited, 1988.

  ISBN 0471796905 LRDC Product Order Number 262030

- Stewart, James. *Single Variable Calculus.* Second edition, Pacific Grove, CA: Brooks/Cole Publishing, 1991.

  ISBN 0534145329 LRDC Product Order Number 262048

The first two resources have been approved as basic student learning resources, and are listed in the course of studies. The third resource has been approved as an authorized teaching resource, but may be used profitably as a student resource with advanced classes aiming for such programs as International Baccalaureate and Advanced Placement. It is not listed in the course of studies. All three are available from the Learning Resources Distributing Centre (LRDC).

In addition, tapes 1 through 11 of the *Catch 31* video series also cover the course material.

Catch 31. Edmonton, AB: ACCESS Network, 1991. [video]

Program 1: *History, Background and Limits*
2: *The Derivative from First Principles: The Power Rule*
3: *The Sum Rule: The Chain Rule*
4: *Product and Quotient Rules*
5: *Problems and Graph Sketching*
6: *Maxima and Minima*
7: *Distance, Velocity and Acceleration*
8: *Derivatives for Relations*
9: *Related Rates*
10: *Integration*
11: *Areas Under or Between Curves*

Neither the course of studies nor the resources selected define the sequence of instruction of the Mathematics 31 topics. Decisions on the sequence of instruction are made at the local level.

CHART 1 THROUGH CHART 4

Charts 1 to 4 following, give the correlations between the course of studies pages and the resource sections. The first column gives the page numbers in the course of studies and the subsequent columns give the sections from the text resources or the video titles.

The resource *Calculus: A First Course* covers all the material in the required section of Mathematics 31, and all the material in all the electives, except for calculus theorems. It generally follows a level consistent with the standards of the Mathematics 31 course, if the Problems Plus sections are used to explore concepts at the standard of excellence.

This resource does not follow the order of the course of studies, as the precalculus is taught on an as needed basis. Trigonometry is dealt with as a complete unit after derivative theorems, with the precalculus of trigonometry covered in detail. When integration is covered, in Chapter 9 to Chapter 11, there is free use of trigonometric and exponential functions from the start.

The resource *Calculus: A Problem-Solving Approach* covers all the material in the required section of Mathematics 31, and all the material in all the electives, except for calculus theorems. The approach used is very numerical and, if these numerical sections are covered, then all the requirements of the elective on numerical methods will have been met. If the sections on numerical methods are omitted, there is some loss of continuity in the treatment of topics. The text generally follows a level consistent with the standards of the Mathematics 31 course, if the later examples in each section are used to explore concepts at the standard of excellence.

This resource does not follow the order of the course of studies, as the precalculus is taught on an as needed basis. Trigonometry is dealt with as a complete unit after derivative theorems, with the precalculus of trigonometry covered in detail. When integration is covered, in Chapter 14 to Chapter 19, there is free use of trigonometric and exponential functions from the start.

The resource *Single Variable Calculus* covers all the material in the required section of Mathematics 31, and all the material in all the electives. It generally follows a level consistent with the standard of excellence of the Mathematics 31 course, as considerable emphasis is placed on the subtleties of limits, derivatives and integrals. Some of the material exceeds the rigour requirements of Mathematics 31, even at the standard of excellence.

This resource generally follows the order of the course of studies, as the precalculus, both algebraic and trigonometric, is presented as an independent section at the beginning. Trigonometry is dealt with as a complete unit, after derivative theorems. When integration is covered in Chapter 4 and Chapter 5, there is free use of trigonometric functions, but exponential functions are covered later, in Chapter 6.

The *Catch 31* videos were originally designed for the old Mathematics 31 course, but the 11 videotapes are still relevant to the new course. However, there is no use made of either trigonometric or exponential functions, and more use is made of the $D_x y$ notation, rather than the $f'(x)$ notation, for the derivative.

CHART 5 THROUGH CHART 8

Charts 5 to 8 following, give the correlations between the text resources, the video resource and the course of studies. In charts 5 through 7 the columns are in pairs: the first giving the section number of the text resource and the second giving the page numbers in the course of studies.

CHART 1: CORRELATION FOR STEWART ET AL., *CALCULUS: A FIRST COURSE*

Course of Studies Pages	Topic	*Calculus: A First Course* Chapter/Section
10–11	algebra of functions	Review and Preview to 2, Review and Preview to 4, Review and Preview to 5, Review and Preview to 6, 4.1, 6.1, 6.2, 6.3, 6.4, 6.5
12–13	transformations	Review and Preview to 5, Review and Preview to 6, Review and Preview to 7, 4.1
14–15	equivalent forms	Review and Preview to 1
16–17	limits	1.2, 1.3, 1.6, 1.7, 7.1
18–19	derivatives	1.1, 1.4, 2.1, 2.2, 2.3
20–21	derivative theorems	Review and Preview to 9, 2.4, 2.5, 2.6, 2.7, 2.8
22–23	trigonometric derivatives	7.2, 7.3, 7.4
24–25	maxima and minima, curve sketching	4.2, 4.3, 4.4, 5.1, 5.2, 5.3, 5.4, 5.5, 5.6, 7.4
26–27	related rates	3.5, 7.4
28–29	antiderivatives	9.1, 9.2
30–31	definite integrals	Review and Preview to 10, 10.4, 11.1
32–33	integral theorems	10.1, 10.2, 11.2
34–35	distance, velocity, acceleration	1.5, 3.1, 3.2, 9.3
36–37	exponential and logarithmic derivatives and integrals	8.1, 8.2, 8.3, 8.4, 8.5, 9.4, 9.6, 10.3
38–39	numerical methods	3.6, 10.5
40–41	volumes of revolution	11.7
42–43	physics applications	3.3, 9.7
44–45	biology applications	8.5, 9.4, 9.6, 10.3
46–47	economics applications	3.4, 4.5
48–49	calculus theorems	11.2
50–51	methods of integration	11.3, 11.4, 11.5, 11.6

CHART 2: CORRELATION FOR REID ET AL., *CALCULUS: A PROBLEM-SOLVING APPROACH*

Course of Studies Pages	Topic	*Calculus: A Problem-Solving Approach* Chapter/Section
10–11	algebra of functions	1.A, 1.B, 8.F, 9.1, 9.2, 9.3, 9.4
12–13	transformations	1.B
14–15	equivalent forms	4.1, 4.2, 4.3
16–17	limits	2.1, 2.2, 2.3, 5.1, 10.1
18–19	derivatives	3.1, 3.2, 3.3
20–21	derivative theorems	3.3, 4.1, 4.2, 4.3, 4.4
22–23	trigonometric derivatives	10.2, 10.3
24–25	maxima and minima, curve sketching	5.1, 5.2, 5.3, 5.4, 6.1, 6.2, 10.3
26–27	related rates	7.1, 8.1, 8.2
28–29	antiderivatives	14.1, 14.2, 14.3, 15.1, 15.4
30–31	definite integrals	15.H
32–33	integral theorems	7.2, 7.3, 16.1, 16.2, 16.3
34–35	distance, velocity, acceleration	15.2, 15.3
36–37	exponential and logarithmic derivatives and integrals	12.1, 12.2, 12.3, 12.4, 12.5, 14.4
38–39	numerical methods	2.4, 3.5, 4.D, 16.1, 16.2, 16.5
40–41	volumes of revolution	19.1, 19.2
42–43	physics applications	none
44–45	biology applications	12.1, 12.2, 12.3, 12.4, 12.5, 14.4
46–47	economics applications	5.E, 6.3
48–49	calculus theorems	none
50–51	methods of integration	17.1, 17.2, 17.3, 17.4, 17.5, 18.2

CHART 3: CORRELATION FOR STEWART, *SINGLE VARIABLE CALCULUS*

Course of Studies Pages	Topic	*Single Variable Calculus* Chapter/Section
10–11	algebra of functions	Review and Preview 1, Review and Preview 4, Review and Preview 5, Review and Preview 6, Appendix B
12–13	transformations	Review and Preview 2, Review and Preview 4, Review and Preview 5, Review and Preview 6, 6.3
14–15	equivalent forms	Appendix A
16–17	limits	Review and Preview 7, 1.2, 1.3, 1.5, 2.4
18–19	derivatives	1.1, 1.6, 2.1, 2.2
20–21	derivative theorems	2.2, 2.3, 2.5, 2.6, 2.7
22–23	trigonometric derivatives	2.4,
24–25	maxima and minima, curve sketching	3.1, 3.3, 3.4, 3.5, 3.6, 3.7, 3.8
26–27	related rates	2.8
28–29	antiderivatives	3.10
30–31	definite integrals	4.1, 4.2, 4.3
32–33	integral theorems	4.5, 5.1
34–35	distance, velocity, acceleration	1.6
36–37	exponential and logarithmic derivatives and integrals	6.1, 6.2, 6.3, 6.4, 6.5, 6.7
38–39	numerical methods	2.10, 7.8, Appendix D
40–41	volumes of revolution	5.2, 5.3
42–43	physics applications	5.4, 5.5
44–45	biology applications	6.1, 6.2, 6.4, 6.7
46–47	economics applications	3.9
48–49	calculus theorems	1.5, 3.2, 5.5, Appendix C
50–51	methods of integration	4.6, 7.1, 7.2, 7.3, 7.4, 7.6

CHART 4: CORRELATION FOR ACCESS NETWORK, *CATCH 31* VIDEOS

Course of Studies Pages	Topic	*Catch 31* Videos
10–11	algebra of functions	none
12–13	transformations	none
14–15	equivalent forms	none
16–17	limits	1
18–19	derivatives	2, 3
20–21	derivative theorems	3, 4, 8
22–23	trigonometric derivatives	none
24–25	maxima and minima, curve sketching	5,6
26–27	related rates	9
28–29	antiderivatives	10
30–31	definite integrals	10
32–33	integral theorems	10, 11
34–35	distance, velocity, acceleration	7
36–37	exponential and logarithmic derivatives and integrals	none
38–39	numerical methods	none
40–41	volumes of revolution	none
42–43	physics applications	none
44–45	biology applications	none
46–47	economics applications	none
48–49	calculus theorems	none
50–51	methods of integration	none

CHART 5: REVERSE CORRELATION FOR STEWART ET AL., *CALCULUS: A FIRST COURSE*

Calculus: A First Course Section Number	Course of Studies Pages	*Calculus: A First Course* Section Number	Course of Studies Pages	*Calculus: A First Course* Section Number	Course of Studies Pages
Review and Preview to 1	14 and 15	Review and Preview to 4	10 and 11	Review and Preview to 8	none
1.1	18 and 19	4.1	10 and 11, 12 and 13	8.1	36 and 37
1.2	16 and 17	4.2	24 and 25	8.2	36 and 37
1.3	16 and 17	4.3	24 and 25	8.3	36 and 37
1.4	18 and 19	4.4	24 and 25	8.4	36 and 37
1.5	34 and 35	4.5	46 and 47	8.5	36 and 37, 44 and 45
1.6	16 and 17	4.6	none	8.6	none
1.7	16 and 17	4.7	none	8.7	none
1.8	none			8.8	none
1.9	none	Review and Preview to 5	10 and 11, 12 and 13		
		5.1	24 and 25	Review and Preview to 9	20 and 21
Review and Preview to 2	10 and 11	5.2	24 and 25	9.1	28 and 29
2.1	18 and 19	5.3	24 and 25	9.2	28 and 29
2.2	18 and 19	5.4	24 and 25	9.3	34 and 35
2.3	18 and 19	5.5	24 and 25	9.4	36 and 37, 44 and 45
2.4	20 and 21	5.6	24 and 25	9.5	none
2.5	20 and 21	5.7	none	9.6	36 and 37, 44 and 45
2.6	20 and 21	5.8	none	9.7	42 and 43
2.7	20 and 21			9.8	none
2.8	20 and 21	Review and Preview to 6	10 and 11, 12 and 13	9.9	none
2.9	none	6.1	10 and 11		
2.10	none	6.2	10 and 11	Review and Preview to 10	30 and 31
		6.3	10 and 11	10.1	32 and 33
Review and Preview to 3	none	6.4	10 and 11	10.2	32 and 33
3.1	34 and 35	6.5	10 and 11	10.3	36 and 37, 44 and 45
3.2	34 and 35	6.6	none	10.4	30 and 31
3.3	42 and 43	6.7	none	10.5	38 and 39
3.4	46 and 47			10.6	none
3.5	26 and 27	Review and Preview to 7	12 and 13	10.7	none
3.6	38 and 39	7.1	16 and 17		
3.7	none	7.2	22 and 23	Review and Preview to 11	none
3.8	none	7.3	22 and 23	11.1	30 and 31
		7.4	22 and 23, 24 and 25, 26 and 27	11.2	32 and 33, 48 and 49
		7.5	none	11.3	50 and 51
		7.6	none	11.4	50 and 51
		7.7	none	11.5	50 and 51
		7.8	none	11.6	50 and 51
				11.7	40 and 41
				11.8	none
				11.9	none

CHART 6: REVERSE CORRELATION FOR REID ET AL., *CALCULUS: A PROBLEM-SOLVING APPROACH*

Calculus: A Problem-Solving Approach Section Number	Course of Studies Pages	*Calculus: A Problem-Solving Approach* Section Number	Course of Studies Pages	*Calculus: A Problem-Solving Approach* Section Number	Course of Studies Pages
1.A	10 and 11	8.1	26 and 27	15.1	28 and 29
1.B	10 and 11, 12 and 13	8.2	26 and 27	15.2	34 and 35
1.C	none	8.3	none	15.3	34 and 35
		8.F	10 and 11	15.4	28 and 29
2.1	16 and 17			15.5	none
2.2	16 and 17	9.1	10 and 11	15.H	30 and 31
2.3	16 and 17	9.2	10 and 11		
2.4	38 and 39	9.3	10 and 11	16.1	32 and 33, 38 and 39
2.5	none	9.4	10 and 11	16.2	32 and 33, 38 and 39
		9.5	none	16.3	32 and 33
3.1	18 and 19			16.4	none
3.2	18 and 19	10.1	16 and 17	16.5	38 and 39
3.3	18 and 19, 20 and 21	10.2	22 and 23	16.6	none
3.4	20 and 21	10.3	22 and 23, 24 and 25		
3.5	38 and 39	10.4	none	17.1	50 and 51
3.6	none			17.2	50 and 51
		11.1	none	17.3	50 and 51
4.1	14 and 15, 20 and 21	11.2	none	17.4	50 and 51
4.2	14 and 15, 20 and 21	11.3	none	17.5	50 and 51
4.3	14 and 15, 20 and 21	11.G	none	17.6	none
4.4	20 and 21			17.I	none
4.5	none	12.1	36 and 37, 44 and 45		
4.D	38 and 39	12.2	36 and 37, 44 and 45	18.1	none
		12.3	36 and 37, 44 and 45	18.2	50 and 51
5.1	16 and 17, 24 and 25	12.4	36 and 37, 44 and 45	18.3	none
5.2	24 and 25	12.5	36 and 37, 44 and 45	18.J	none
5.3	24 and 25	12.6	none		
5.4	24 and 25			19.1	40 and 41
5.5	none	13.1	none	19.2	40 and 41
5.E	46 and 47	13.2	none	19.3	none
		13.3	none	19.4	none
6.1	24 and 25			19.5	none
6.2	24 and 25	14.1	28 and 29		
6.3	46 and 47	14.2	28 and 29	20.1	none
6.4	none	14.3	28 and 29	20.2	none
		14.4	36 and 37, 44 and 45	20.3	none
7.1	26 and 27	14.5	none	20.4	none
7.2	32 and 33	14.6	none	20.K	none
7.3	32 and 33				
7.4	none				

CHART 7: REVERSE CORRELATION FOR STEWART, *SINGLE VARIABLE CALCULUS*

Single Variable Calculus Section Number	Course of Studies Pages	*Single Variable Calculus* Section Number	Course of Studies Pages
Review and Preview. 1	10 and 11	4.1	30 and 31
Review and Preview. 2	12 and 13	4.2	30 and 31
Review and Preview. 3	none	4.3	30 and 31
Review and Preview. 4	10 and 11, 12 and 13	4.4	none
Review and Preview. 5	10 and 11, 12 and 13	4.5	32 and 33
Review and Preview. 6	10 and 11, 12 and 13	4.6	50 and 51
Review and Preview. 7	16 and 17		
		5.1	32 and 33
1.1	18 and 19	5.2	40 and 41
1.2	16 and 17	5.3	40 and 41
1.3	16 and 17	5.4	42 and 43
1.4	none	5.5	42 and 43, 48 and 49
1.5	16 and 17, 48 and 49		
1.6	18 and 19, 34 and 35	6.1	36 and 37, 44 and 45
		6.2	36 and 37, 44 and 45
2.1	18 and 19	6.3	12 and 13, 36 and 37
2.2	18 and 19, 20 and 21	6.4	36 and 37, 44 and 45
2.3	20 and 21	6.5	36 and 37
2.4	16 and 17, 22 and 23	6.6	none
2.5	20 and 21	6.7	36 and 37, 44 and 45
2.6	20 and 21	6.8	none
2.7	20 and 21	6.9	none
2.8	26 and 27	6.10	none
2.9	none		
2.10	38 and 39	7.1	50 and 51
		7.2	50 and 51
3.1	24 and 25	7.3	50 and 51
3.2	48 and 49	7.4	50 and 51
3.3	24 and 25	7.5	none
3.4	24 and 25	7.6	50 and 51
3.5	24 and 25	7.7	none
3.6	24 and 25	7.8	38 and 39
3.7	24 and 25	7.9	none
3.8	24 and 25		
3.9	46 and 47	Appendix A	14 and 15
3.10	28 and 29	Appendix B	10 and 11, 22 and 23
		Appendix C	48 and 49
		Appendix D	all sections of course

CHART 8: REVERSE CORRELATION FOR ACCESS NETWORK, *CATCH 31* VIDEOS

Catch 31 Video Number	*Catch 31* Title	Course of Studies Pages
1	*History, Background and Limits*	16 and 17
2	*The Derivative from First Principles: The Power Rule*	18 and 19
3	*The Sum Rule: The Chain Rule*	18 and 19, 20 and 21
4	*Product and Quotient Rules*	20 and 21
5	*Problems and Graph Sketching*	24 and 25
6	*Maxima and Minima*	24 and 25
7	*Distance, Velocity and Acceleration*	34 and 35
8	*Derivatives for Relations*	20 and 21
9	*Related Rates*	26 and 27
10	*Integration*	28 and 29, 30 and 31, 32 and 33
11	*Areas Under or Between Curves*	32 and 33

TECHNOLOGY AND MATHEMATICS 31

The following assumptions have been made in the Mathematics 31 Course of Studies on the point of technology.

- First, the decision on technology should be made at the local level, bearing in mind the environment in Alberta post-secondary institutions. Student needs should be paramount.

- Second, some numerical work should be done by all students to establish the concepts of limit, derivative, integral and related rate. Exact values are not possible, but even with a hand-held calculator, good approximations are possible.

- Third, all students should be able to use the classical methods of curve sketching. Students with high-powered graphing utilities will use them first, and then use the classical methods to find the exact nature of any critical areas on the curve, or restrictions on the domain and the range.

- Fourth, students should be aware of the limitations of any numerical solution, and be able to evaluate the reliability of a numerical solution.

At present, the pace of software development is rapid, and new programs are capable of performing such tasks as symbol manipulation, algebraic simplification, and the evaluation of derivatives and antiderivatives in symbolic form. Some programs, such as Mathematica™ and Maple™, can explore calculus concepts at different levels of rigour. To discuss the levels of technology commonly encountered in school and post-secondary classrooms, the following classifications can be used.

Level 1: All technology banned, even simple calculators. Exact values for trigonometric functions of special angles to be memorized. This level is commonly used for university examinations.

Level 2: Scientific calculators allowed, but not the graphing capabilities. This level is fairly common in Alberta schools.

Level 3: Scientific calculators allowed, and graphing utilities (computer and calculator) used in selected areas of the course. This level is used in many institutions in other provinces, especially Ontario, in their Ontario Academic Course (Calculus).

Level 4: Computer productivity software in common use, with high emphasis on numerical solutions, using spreadsheet programs, such as Excel™ and ClarisWorks™. This level is one employed by schools operating in a high-tech environment. Graphing programs often have animation to emphasize the dynamic nature of graphs.

Level 5: Full use is made of algebraic symbol-manipulating programs, such as Mathematica™, Maple™, Calculus T/L II™ and MathCAD™. At present, there are conflicting beliefs whether or not students can learn these ultrahigh-level languages at the same time as learning the fundamental concepts of calculus. There is no doubt as to the utility of these programs, once the fundamental concepts of calculus have been mastered.

The new resources approved for use in Mathematics 31 support a course based on any of Level 1 to Level 4. However, they do not support Level 5. Any such course would require the use of special software, with text and laboratory materials integrated into that software. So far, such resources are in a developmental state only. When one or more such resources are published in a suitable form, they may be approved and a Level 5 approach to Mathematics 31 could be taught. In addition, a Level 5 approach would require students to have individual access to sophisticated hardware (Macintosh™ SE or higher, MS-DOS 486/25 or higher with Windows™) for 3 to 6 hours per week for laboratory activities. In detail, the approved resources can be described as follows.

> *Zap-A-Graph,* Release 4.0, or its equivalent Windows™ application, provides all the graphing needed. There are two problems with this release: displayed equations tend to be written in unsimplified form, especially under the derivative command, and there is no easy zoom command for detailed investigation of small areas of a curve.
>
> Stewart et al., *Calculus: A First Course,* is a classical low-technology approach to first-year calculus. All it assumes is the use of a standard scientific calculator.
>
> Reid et al., *Calculus: A Problem-Solving Approach,* does more numerical work, including a chapter on differentials. It uses BASIC in its programs, not spreadsheet technology. The programs cited are relatively simple and easily adapted to spreadsheet technology.
>
> Stewart, *Single Variable Calculus*, has an appendix "Lies My Calculator or Computer Told Me," which is essential reading.

None of the resources is suitable for a Level 5 technology approach, and schools going this route should select another resource and have it approved by local school board motion.

There are many graphing utility programs on the market, at a wide range of prices, so purchases should be guided by their usefulness. The minimum requirements should be:

1. ability to operate in define mode and in transform mode. Sometimes a curve has its equation known (use define mode), and sometimes the equation is unknown, but its relation to a previous function is known (use transform mode). This relation may be geometric; e.g., rotate $\pi / 4$ counterclockwise, or algebraic; e.g., reciprocal, derivative or absolute value

2. capability of adding, subtracting, multiplying, dividing, composing and inverting functions. Keying sequences or menu selections will vary from utility to utility

3. user option for displaying the equation of the function or relation being graphed

4. user option for root-searches, at least for the equation $f(x) = 0$.

DATE DUE SLIP

F255

University of Alberta Library
0 1620 0242 8215